FRACTAL CITIES

άγεωμέτρητοζ μηδεὶζ εχσίτω
Let no one enter who does not know geometry

Anon, Inscription on the door of Plato's Academy
at Athens, although often accredited to Plato himself

Elias Philosophus, *In Aristotelis Categoria Commentaria, 118.18*
(A. Busse Ed. *Comm. in Arist. Greaca*, Berlin, 1900, XVIII, i.)
From *The Oxford Dictionary of Quotations*, 1979, 3rd Edition.

FRACTAL CITIES

A Geometry of Form and Function

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Preface

Fractal geometry was invented almost single handedly by Benoit Mandelbrot over a thirty year period from 1950. It burst onto the academic stage 10 or so years ago through its ability to generate highly realistic computer graphic scenes of the natural world, which were popularized in Mandelbrot's (1983) remarkable book. At the same time, it was becoming central to the reawakening of interest in the science of form in the fields of physics and biology. In mathematics too, the new geometry was instrumental in visualizing solution spaces of dynamic systems whose behavior could no longer be regarded as smooth, but discontinuous and chaotic. The key insight emanating from these diverse origins revolves around the idea that the world is chaotic, discontinuous, irregular in its superficial physical form but that beneath this first impression lies an order which is regular, unyielding and of infinite complexity. But this is an order that has been simplified away in terms of the continuous and the smooth in all previous attempts at scientific understanding. In this sense, the world must now be seen as largely fractal. Non-fractal or smooth renditions of it and the science that accompanies this is thus the science of the special case.

It is hard to impress the importance of this insight but in the last decade, fractal geometry has found its way into many sciences and arts. Everything from Stephen Hawking's theories of the universe to George Lucas's *Star Wars* movies and popular novels such as Michael Crichton's *Jurassic Park* seem to be touched by fractals, while statements of its importance to science and modern society abound. John A. Wheeler says that:

> "No one is considered scientifically literate today who does not know what a Gaussian distribution is, or the meaning and scope of the concept of entropy. It is possible to believe that no one will be considered scientifically literate tomorrow who is not equally familiar with fractals"

while Hugh Kenner describes the field as being

> ". . .as big a picture as this century has seen".

James Gleick in his book *Chaos* says that: ". . .twentieth-century science will be remembered for just three things: relativity, quantum mechanics and chaos", and this includes the geometry of chaos – fractals. The superlatives continue and readers could be forgiven for throwing up their hands in horror at yet more hype about fractals.

The time has come however for these ideas to be absorbed into the mainstream of science. Recently, with the dust settling, many applications of fractal geometry are being developed in diverse fields and researchers are finding that fractal ideas have been part of our consciousness, certainly from the time of the Italian Renaissance, but probably as far back as the Greeks, and maybe before. What fractal geometry does is to provide a framework for tying together previously unconnected and diverse concepts, thus enabling the assembly of a 'bigger' picture. Cities yield some of the best examples of fractals. For generations, architects and planners have attempted to impose a simple, smooth, visual order on cities in the belief that such order counters the disorder and dysfunction which cities reveal when they develop 'naturally'. All the great Utopias from Plato onwards have sought to impose the geometry of Euclid on the city as an example of man's triumph over nature. In this way, art has been separated from science. But this viewpoint has always been opposed in some measure. In the last 50 years, with the realization that social and economic order belies the physical form of cities, the idea that the naturally or organically growing city is optimal in countless ways which we have hitherto ignored, has grown in strength. In short, our view now about the shape and form of cities is that their irregularity and messiness is simply a superficial manifestation of a deeper order. And as we will argue here, fractal geometry has much to say about this.

This book presents an initial attempt to apply fractal geometry to cities. In fact, we go beyond this and argue that cities are fractal in form, and that much of our pre-existing urban theory is a theory of the fractal city. As befits a beginning, this book is very much an introductory statement and, as we argue throughout, these ideas are simply crude snapshots of a much bigger picture which we hope others will steer their research towards. In terms of theory, we show here that the architect's physical determinism concerning the city can be captured and elaborated in terms of fractals while the geographer's concern for the economic theory of location is entirely consistent with the use of fractal ideas. We live in an era when physical determinism is still disreputable as architects and city planners seek to minimize the impact of designs which manifestly interfere with the social and economic fabric of cities in countless unanticipated and undesirable ways. But physical form does determine the quality of life in cities. We see fractal geometry as providing a new hope for understanding the power of determinism, as well as new methods for enabling the synthesis of urban density with central place theory, new ways of visualizing the impact of human decision-making on cities, and perhaps most of all, new goals for achieving the good society through manipulating and planning city form.

One of the central themes of this book is based on the need to 'visualize' complex spatial phenomena, in our case cities. Visualization has come hard on the heels of developments in computer graphics during the 1980s and now forms a major force in advancing science whose systems are spatial in some sense, and whose form reveals infinite complexity. Urban theory has not advanced to the extent it might because the intrinsic complexity and extensiveness of data has been difficult to grasp just as the results of modeling such complexity have been difficult to absorb and evaluate. Computer graphics as scientific visualization thus blazes a path within science from which we can learn how to make progress through studying examples and applications from many diverse fields. Although our focus is on cities, we see the examples in this book as being as relevant to biology and astronomy, say, as to our own more limited domain just as we see the entire field of visualization as helping us to fashion computer graphics to our own problems. We thus consider that creative advances in visualization are unlikely to be restricted to any particular field, metaphors and analogies between one field and another being the way forward. In this sense, our text is part picture book from which those interested in graphics and visualization might draw some inspiration. It is through pictures that our ideas can be most readily and clearly articulated, and there is perhaps something about the icons we employ that might encourage the sort of interdisciplinary research which is central to our own field.

Our book also provides a gentle introduction to fractal geometry and it is our hope that through this medium, the more general scientific reader will find much of interest here. Cities, as we said earlier, are fine example of fractals and we believe that this book could be used in courses which teach their rudiments, especially to those whose mathematics is at the 'beginning calculus' level. Moreover, it is our view that fractals are as much a part of the artificial as the natural world, that they are writ large in social and well as physical systems. Thus we hope that our treatment might be of use to social scientists whose concern in some way is with space, and, of course, to geographers, urban economists, planners, engineers, designers, mathematicians, and computer scientists.

Let us say a few words about the origins of the book. For one of us (Batty), fractals go back a long way to the 1960s through ideas in location theory involving power laws in rank-size and central place theory. In fact, the first edition of Peter Haggett's *Locational Analysis in Human Geography* published in 1965 was full of references to the effect of form on location and cities. Mandelbrot's work in this area was known to us at that time, and again in 1974, Michael Dacey pointed out its relevance to us. Mandelbrot's first edition of his book *Fractals: Form, Chance, and Dimension* published in 1977 escaped us but in 1981, Lionel March mentioned it and from then on, fractals seemed to be always with us. Of course, it was clearly the emergence of computer graphics which fueled the fire and by the time Mandelbrot published his second English edition *The Fractal Geometry of Nature* in 1982, the movement was in full swing.

It is unlikely that we would have begun work in this area unless one us (Batty again) had not attempted to train himself in computer graphics in the early 1980s. Almost accidentally, an attempt at generating Mandelbrot's fractal planet on a very small (32K RAM) microcomputer yielded a remarkable and startling realism (as shown in Plate 3.5). From then on we were hooked. Paul Longley began his university career in Bristol in 1977 in the same year Peter Haggett produced the much enlarged second edition of his book which captured much of the synthesis forged within locational analysis during the 1960s and 1970s. After lecturing in the Universities of Karlsruhe and Reading, he joined Michael Batty in Cardiff in 1984 and as part of a project to develop computers in city planning and building on our past work in urban models, we cultivated these interests from 1986 on.

Many people have helped us with this book. Its production would not have been possible without the skills and efforts of our technical staff. Tra-

cey Dinnock drew some of the figures, Martin Morris and Nathan Webster took many of the photographs, and Andrew Edwards assembled hardware and software when we were both associated with the Department of City and Regional Planning at the University of Wales in Cardiff. Tony Philpott and Simon Godden have helped us more recently with the final production of the book in the Department of Geography at the University of Bristol. Kate Brewin, our editor at Academic Press, has been most supportive.

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In Buffalo itself, Mike Woldenberg, the source of all historical wisdom on fractals and allometry, continues to engage us in vibrant discussion while David Mark injected us with an equally influential skepticism. Yichun Xie helped with C and UNIX and was responsible for the computer analysis of the North East US cities examples in Chapter 7. Pierre Frankhauser of the Universite de Franche-Comte, Besancon, read the complete manuscript and gave us comments at a late stage when we met him in Buffalo and Budapest. We thank all these colleagues and friends, and many who go unremarked, for their help. In as controversial a set of applications as these, they bear no responsibility other than sharpening us to the criticism.

This book has been a long time in the making. It was begun in earnest when Paul Longley spent a semester of study leave at Buffalo in late 1991. A lot of the earlier work was done when we in the University of Wales at Cardiff in one of the most dynamic and turbulent environments we are ever likely to encounter. It was in that place that this started and at times, we wondered whether we and the research would survive it. But it is a testament to the power of ideas, especially good ones such as fractals, that they (and we) did do so, and nothing would be more satisfying to us than

if just a few of these were to be picked up by others in our field and made still more applicable.

Finally, we thank our respective families, Sue and Daniel, and Mandy, who cheerfully tolerated the writing of this book, amidst important domestic events such as countless houses and house moves, GCSEs, and marriage. We dedicate it to them.

Michael Batty **Paul Longley**

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Figures 24 and 25 from Keeble, L. (1959) *Principles and Practice of Town and Country Planning*, The Estates Gazette, London; figures 153, 163,and 193 from Kostof, S. (1991) *The City Shaped: Urban Patterns and Meanings Throughout History*, Little, Brown and Company, Boston, MA; the figures on pages 21 and 22 from McGuire, M. (1991) *An Eye For Fractals: A Graphic and Photographic Essay*, Addison-Wesley Publishing Company, New York; figures 4.11, 4.77, 5.11, 5.12, 5.13, 5.14, 5.15, 5.17, 5.32 and 7.15 from Morris, A. E. J. (1979) *History of Urban Form: Before the Industrial Revolutions*, Longmans Scientific and Technical, London; New York; figures 84, 119, 289, and 291 from Reps, J. W. (1965) *The Making of Urban America: A History of City Planning in the United States*, Princeton University Press, Princeton, NJ; the figure on page 259 of Vance, J. E. (1990) *The Continuing City: Urban Morphology in Western Civilization*, The Johns Hopkins University Press, Baltimore, MD.

We thank The Glamorgan Archive Service for permission to reprint the following maps of Cardiff: by John Speed 1610, Cardiff in the late 1830s by John Wood, Cardiff in 1851, and District Plan of Cardiff and Penarth 1869.

We thank Ian Braecken for plates 7.1 and 7.2, and Keith Horsfield for the plaster cast of the lung reproduced in Fig. 2.15(a). These plates and figures were made available especially for this book.

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Introduction

Fractal geometry will make you see everything differently. (Barnsley, 1988a, p. 1.)

Our view of cities is changing. One hundred years ago, cities were understood and planned as physical artifacts with predominant concern for their architecture and their aesthetics. It was widely agreed that their physical form was the ultimate determinant of their social and economic functioning, and the quality of life therein. As the 20th century wore on, this view weakened as attention turned to their economic structure and the efficiency of their organization, but until the 1960s, the physical viewpoint remained central, notwithstanding a gradual shift from aesthetic concerns to locational efficiency.

Prevailing views and approaches then changed dramatically. During the last 25 years, attention has been focussed upon the institutional structure of cities, on social processes and class conflict, on urban poverty and deprivation, on their diverse roles in the local and global economy. After a century of sustained effort at their understanding, our knowledge is still partial and fragmentary, based on a kaleidoscope of viewpoints and ideologies. What, however, is widely accepted, perhaps a little reluctantly by some, is that cities are mirrors and microcosms of society and culture at large, with every viewpoint contributing something to their understanding. Yet in this quest, there have been few successes in evolving our understanding from one approach to another. Consequently, the physicalism based on the idea of cities as being 'architecture-writ-large' cannot be easily related to the theory of cities as social or economic or institutional systems – social processes are not easy to relate to spatial form – and thus our current understanding is overwhelmed by their complexity and diversity.

Yet there is a curious paradox in all of this. We know instinctively that the physical form of cities is the ultimate result of a multitude of social and economic processes, constrained and shaped by the geometry of the natural and man-made world. We know that urban problems are manifest in the first instance in physical and spatial terms. We also know that many, if not most, of the instruments we have at our disposal for designing better cities are physical in form and intent. But there is still no widespread consensus as to the importance of form, geometry, layout, and configuration which characterize the physical city. Physicalism has fallen out of fashion in many fields in the last 50 years. This now appears to be changing too, and there is new hope that a more profound understanding of physical form in science generally is about to emerge. In this, there are important implications for how we might develop a new understanding of the form of cities. The challenge we set ourselves in this book is to address these possibilities.

These movements are clearly a part of more fundamental secular trends in the way we fashion knowledge and science and apply it in society. One hundred years ago, there was great optimism, great certainty that science and technology would lead us to the good society. Today we know different. That certainty and optimism has dissolved and has been replaced by a degree of uncertainty and a skepticism as to whether our science can ever yield the answers. This change is not only a social effect of the difficulties realized in the application of technology, but has also emerged from science itself. The quest for comprehensiveness through systems thinking which came to dominate science by the mid-20th century led to the profound discovery that even the simplest physical systems can admit uncertainty in their predictions. The emergence of catastrophe theory, and now chaos, first in mathematics and then in countless applied areas, is testament to this change. The current concern for complexity and diversity, for micro–macro and local–global properties of systems, for treating the world simultaneously on many levels, is reinforcing these trends, while the gradual realization that there is order in chaos, is taking science and mathematics to new levels of abstraction, far away from the most elemental of human intuitions from whence springs the original thirst for knowledge.

Fractal geometry is part of this change. For the first time, a formal framework for the geometry of the real world rather than for its abstraction into pure mathematics has been established which enables us to understand order and regularity in what, at first sight, appears irregular and disordered. Rather than starting with function and progressing to form, fractal geometry enables us to search out functions and processes which give rise to the man-made and natural patterns we observe in the real world, thus helping us not only to describe and understand reality a little better but to progress our forecasts and predictions of how the real world might evolve. Geometry is no longer conceived in terms of straight lines – the geometry of Euclid – but can now admit irregularity without abandoning continuity, thus relaxing the severity of mathematics to encompass what is natural and essential. The application of this geometry to cities is immediate. Planned cities are cast in the geometry of Euclid but by far the majority, those which are unplanned or planned less, show no such simplicity of form. Moreover, all cities contain some organic growth, even planned cities are adapted to their context in more natural ways once the plan comes to be implemented, and in any case, the extent to which human decision-making is ordered or planned is always a matter of degree. In this sense then, all cities show some irregularity in most of their parts and are thus ideal candidates for the application of fractal geometry.

This book is but a beginning in the quest to develop a robust and relevant geometry for the spatial organization of cities. Much of the material we introduce here is speculative and informal, in that it is part of an ongoing research program which we hope will develop and grow as the logic of these arguments is appreciated. We will sketch out how fractal geometry might be applied to cities in many ways; first in terms of visualizing urban form through computer models and computer graphics, and then through the measurement of patterns in real cities and their dynamic simulation. We will illustrate these ideas with hypothetical and real cities and with simulations or models. We will focus on physical forms such as city boundaries, networks, hierarchies, urban texture, and the density of population amongst many structural concepts. We will deal with real, ideal and hypothetical concepts, showing how these might be best applied to yield general insights into urban form and functioning. It is not our intention to provide a blueprint for a new theory of cities. This may come eventually from fractal geometry, but as we continually emphasize, this is merely a beginning.

At the very outset, we must give the reader a brief but cogent idea of fractal geometry and its relevance to cities. Fractals, a term coined by their originator Benoit Mandelbrot (1983), are objects of any kind whose spatial form is nowhere smooth, hence termed 'irregular', and whose irregularity repeats itself geometrically across many scales. In short, the irregularity of form is similar from scale to scale, and the object is said to possess the property of self-similarity or scale-invariance. It is the geometry of such objects which is fractal, and any system which can be visualized or analyzed geometrically, whether it be real or a product of our mathematical imagination, can be fractal if it has these characteristics. We do not wish to preempt our discussion in Chapters 2 and 3 but when we refer to an object as being irregular, we do not mean that it is disordered or chaotic, but that it is not smooth in the sense in which Euclidean geometry articulates the world.

Because fractal objects are not smooth, their geometry poses certain basic conundrums. The classic example is the coastline. Coastlines are never straight, nor do they twist and turn in such a way that they enclose space completely. But on an intuitive level, they are something more than the straight line which has a Euclidean dimension of 1 and something less than the plane in which they exist which has a dimension of 2. We could guess, and in fact we would be correct, that their dimension was between 1 and 2, fractional rather than integral, thus opening up the possibility that the Euclidean dimensions of 1 and 2 are simply extremes or special cases which bound a continuum of fractional or fractal dimensions. In Chapter 2, we will imply that this argument can be generalized to any dimension, and that although coastlines and terrain are fractal in the familiar realm of space–time, fractals exist in higher dimensions, in mathematical space, where their visualization only makes sense to abstract analysis. The conundrums emerge for all fractals whose measurement which is usually based upon the integral part of their fractal dimension, no longer accords to this Euclidean logic. For coastlines, for example, this means that their length in the linear dimension is infinite although the area that they enclose is bounded. More, much more of this, in Chapter 2.

In this book, we will restrict our discussion to fractals which exist in the continuum from points to lines to planes to volumes which are all geometric notions useful in describing the city. Thus our geometry is a literal one and not one abstracted from these real properties of the city. In fact, most of our analysis of the geometry of cities, of urban form, will deal with boundaries and areas. We will not consider the city in its third dimension which might also be regarded as fractal – the skyline of Manhattan for example – although in a sense, the third dimension will enter implicitly when we deal with population densities in later chapters. Our applications to cities also reveal the essential logic of fractal geometry. If fractals are self-similar in that their geometry repeats itself on many levels or scales, it is clear that a stable process or set of processes is operating to form the geometry. For example, coastlines and terrain are determined by geomorphic processes of weathering and erosion. Cities of different sizes are associated with specialization due to economies of scale, and so on. Of course, if we can demonstrate that an object is fractal, this gives us some hope that we can narrow our search for those functions and processes which give rise to different forms.

Cities have quite distinct fractal structure in that their functions are self-similar across many orders or scales. The idea of neighborhoods, districts and sectors inside cities, the concept of different orders of transport net, and the ordering of cities in the central place hierarchy which mirrors the economic dependence of the local on the global and vice versa, all provide examples of fractal structure which form the cornerstones of urban geography and spatial economics. Hierarchical distributions of city size in the form of Pareto or rank-size rules have long been considered the iron laws of spatial systems, while distributions and density profiles inside the city are fractal in nature. We will not attempt to rewrite the entire edifice of urban theory in fractal geometry within this book although we will touch upon such theory at many points. This may disappoint some who feel that these ideas are suggestive enough to demand such a reworking. But ours is a different quest, once again a beginning, a demonstration of what is possible. We hope to lay out some promising trails, but doubtless, we will trace some false ones. And we are not too arrogant to confess that there are many aspects and implications of fractal geometry which lie beyond our expertise and must be taken up by others, with different interests and objectives in mind. Finally there are many aspects of cities which can be interpreted as fractal in the geometric sense, and as yet there is no coherent theory to pull all this diversity together. Nor need there be, for if fractal geometry is to be truly general like any geometry, its applicability to even similar systems of interest will be likewise diverse. In so far as integrated theory exists to which fractal geometry can be applied, this integration will be in terms of the system itself and not its geometry.

The cities, and systems of cities, which we will consider range from the hypothetical to the real. Those which are hypothetical exist as idealized abstractions and are mainly introduced in the earlier chapters where we are concerned with laying the foundations of fractal geometry. These enable us to explore how we might develop appropriate techniques for measuring and modeling fractal distributions. However, most examples in this book are concerned with real cities. From Chapter 5 on, we will develop methods for describing and measuring fractal properties of cities which exist at cross-sections in time, in terms of boundaries to entire cities, and to the land uses which they comprise, as well as the urban textures which reflect their spatial juxtapositioning. We then launch into a study of the fractal city proper, measuring its growth over time, its processes of growth, and ways in which we might model this growth, connecting all these ideas up with urban economic theory in the form of population densities on the one hand, and systems of cities which compose central place theory on the other. Our exposition will begin with the superficial geometry of the city but will then expand to examine its internal functions and its positioning within the wider hierarchy of city sizes.

We will begin in Chapter 1 by reviewing what we know about the shape of cities, about urban form, particularly emphasizing the distinctions between 'planned' or 'regular' cities, and 'organic' or 'irregular' cities. This is a matter of degree, of course, but our argument will establish the case for a fractal geometry of cities, and we will show how the concepts of hierarchy and self-similarity particularly, comprise the structure of knowledge in this domain. Our next three chapters lay the foundations for a fractal theory of cities largely, but not exclusively, based on hypothetical or at least highly abstracted city shapes. In Chapter 2, we outline the rudiments of fractal geometry, presenting classic well-defined 'deterministic' fractals such as space-filling curves, trees and hierarchies of various kinds, emphasizing the appropriate mathematics of shape, scale and dimension, and methods for constructing fractals through recursion and iteration. In Chapter 3, we generalize these ideas to fractals whose properties must be measured statistically, concentrating largely on methods for estimating the fractal dimension of coastlines and terrain, as well as on appropriate techniques for their generation. We then use these techniques to simulate the patterning and texture of land use in a large city with an emphasis on visualization using computer graphics. We take these ideas much further in Chapter 4 where we set up a laboratory for visualizing different urban forms which are possible using these types of model, and finally we apply these ideas to a real example – residential housing location in London.

Chapters 2–4 explain the use of fractals in visualization and in simulation rather than in measurement, but in the next two chapters we change tack to consider much more specific methods of measurement and estimation. In Chapter 5, we rework the classic fractal as the boundary of an urban area – the edge of the city, while in Chapter 6, we extend these ideas to sets of many fractal objects, in this case, those land uses which comprise the elements of the city which exist within its urban boundary. This introduces a key theme into this discussion, namely the distinction between methods for estimating the fractal dimension of a single object, and methods for estimating the dimension of a set of many objects, and the ways these are related. By this point, we have introduced enough theory and method to begin the construction of fully-fledged dynamic models of the city and systems of cities using fractal geometry, and this we accomplish in the rest of this book.

Chapter 7 is one of the most important for there we lay the groundwork for a theory of the fractal city in contrast to the discussion hitherto which has been pitched around the idea of fractal theory applied to cities. We begin by examining the form of several cities around the world, focussing upon their fractal properties in terms of the way their development fills available space. The model we propose builds on the ideas already introduced but is set within the context of the growing city and the way development generates regular gradations of density associated with its space-filling. We then propose a more general model based on the idea of limited diffusion of growth – the so-called diffusion-limited aggregation (DLA) model which is widely used in non-equilibrium physics – and we spend the rest of this chapter fitting it to real examples. We then extend and generalize the model in Chapter 8 to provide a new laboratory for generating a range of different urban forms and we select one which best fits the form of the city of Cardiff, thus providing simple, but nevertheless suggestive, simulations of urban growth and form. In Chapters 9 and 10, we embed these ideas first into the theory of urban population densities and then into central place theory. In Chapter 9, we show how fractals provide a new and enriched view of density theory, implying functional forms different from those used traditionally, but also providing more informed methods for measuring and estimating density functions than those used hitherto. In Chapter 10, we generalize these density and scaling relations to a system of cities, to the central place hierarchy, and show how constraints on their form influence the pattern of settlement at the regional level. In both these chapters, we complete our theory of the fractal city by extending its scaling relations to embrace both density theory and urban allometry.

Our conclusions are brief but directed, mainly pulling together the many threads we have sought to weave but also charting plans for the future. We reflect there upon the purpose of this work. Clearly fractal geometry enables us to link form to function which is our initial motivation, and at least, it shows us how we might approach this quest. What we do not intend here is to suggest models which are immediately applicable in a policy context, for our concern is much more with demonstrating an approach. At one level, this might be adapted to specific contexts, at another, it might simply suggest how we think about cities. Furthermore, we can even argue that fractal methods are more applicable to the generic city than to actual or particular cities, and in this sense, are mainly expository rather than applicable. But we will not take this discussion any further here for these are issues that the reader must decide.

Before we begin, we must say something briefly about the mathematical notation we will use. Throughout the book, we will use a standard set of definitions to refer to geometrical entities: distances, numbers of objects or populations, lines, areas, and densities. The overall linear size of an object will be defined by the variable R and its reference at any explicit point by r; these are typically distances in cities whose size is measured thus. The numbers of objects, usually associated with a system of size R is given by N, in turn referenced in terms of its parts by n. Length of lines and areas are denoted by L and A respectively, while density, defined as N/A, is given as ρ. Each of these variables might be indexed by size or distance r or R to which they refer, or discretely by integers in the range i, j, k, l, m and n depending upon context. With respect to dimension, the fractal dimension is defined as D while the Euclidean or integer dimension is E. Note that we will use d as a measure of discrete distance, not as part of the continuum $r \subset R$, and that the statistical measure of fit or coefficient of determination r^2 should not be confused with size or distance r. In Chapter 10, E_k is also used to define the length of the envelope of an urban area specific to settlement k. All other variables and parameters will not be completely standardized but defined chapter by chapter, although some attempt will be made to make their usage unique. Equations which are numbered are always referred to explicitly in the text by their number; those which are not do not have any lesser significance but are simply not referred to in the text.